

**REVIEW OF “INTERRELATION BETWEEN WEAK  
FRAGMENTS OF DOUBLE NEGATIN SHIFT AND RELATED  
PRINCIPLES” BY MAKOTO FUJIWARA AND ULRICH  
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DANKO ILIK

The paper studies the interrelation between the double negation shift principle for  $\Sigma_1^0$ - and  $\Sigma_2^0$ -formulas ( $\Sigma_1^0$ -DNS,  $\Sigma_2^0$ -DNS) and the *double negation* of the following principles from intuitionistic meta-mathematics: of the limited principle of omniscience ( $\neg\neg$ LPO), of the lesser limited principle of omniscience ( $\neg\neg$ LLPO), of the weak limited principle of omniscience ( $\neg\neg$ WLPO), of Markov’s principle ( $\neg\neg$ MP), of disjunctive Markov’s principle ( $\neg\neg$ MP<sup>∨</sup>), and of two variants of the law of excluded middle for  $\Delta_1^0$ -formulas ( $\neg\neg\Delta_a$ -LEM,  $\neg\neg\Delta_b$ -LEM).

It is shown that only the following implications hold (and the ones that follow from these by transitivity):

- $\Sigma_2^0$ -DNS  $\rightarrow$   $\neg\neg$ LPO,
- $\neg\neg$ LPO  $\rightarrow$   $\neg\neg$ WLPO  $\rightarrow$   $\neg\neg$ LLPO  $\rightarrow$   $\neg\neg$ MP<sup>∨</sup>,
- $\neg\neg$ LPO  $\rightarrow$   $\neg\neg$ MP  $\leftrightarrow$   $\neg\neg\Delta_b$ -LEM  $\rightarrow$   $\neg\neg$ MP<sup>∨</sup>,
- $\neg\neg$ MP<sup>∨</sup>  $\rightarrow$   $\neg\neg\Delta_a$ -LEM, and
- $\neg\neg\Delta_b$ -LEM  $\rightarrow$   $\Sigma_1^0$ -DNS.

While some of the above implications have been known before, establishing a *complete* characterization of the interrelation between the principles – i.e., showing that no other implication between them holds – is the major contribution of this article. The counter-models that establish the non-implications are detailed in Section 4 of the paper.