REVIEW OF "NONSTANDARD FUNCTIONAL INTERPRETATIONS AND CATEGORICAL MODELS" BY AMAR HADZIHASANOVIC AND BENNO VAN DEN BERG (2017)

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The paper concerns the interpretation of principles of non-standard arithmetic via Dialectica interpretation and sheaf toposes.

Section 2 revisits the prior work by van den Berg, Briseid, and Safarik, and defines in detail a version of the Dialectica interpretation which can interpret "herbrandized" versions of nonstandard uniformity (NCR), independence of premise (HIP), the axiom of choice (HAC), as well as the principles of overspill (OS) and underspill (US) for finite sequences, and consequently $LLPO$ and herbrandized generalized Markov's principle. The soundness theorem (Theorem 2.25) allows to show conservativity of $H := E\text{-}HA^\omega + OS + US + NCR + HIP + HAC$ over $E\text{-}HA^\omega$ for at least the internal atomic formulas (the equivalence between a formula ϕ and its interpretation ϕ^D seems to require the full theory H in Theorem 2.27).

The idea of the non-standard Dialectica interpretation is that, instead of a single witnessing candidate, the interpretation keeps a finite list of potential witness-candidates, at least one of which is expected to hold (hence the name "herbrandized").

Section 3 shows that there is a model for H, namely the filter topos of Moerdijk. Moreover, it is shown that this topos also satisfies the transfer rule, important for switching between internal and external quantifiers, as well as the fan theorem for negative formulas.

Section 4 defines another version of the Dialectica interpetation, called uniform Diller-Nahm interpretation. The soundness theorem (Theorem 4.5) shows that uniform Diller-Nahm interprets the non-herbrandized versions of nonstandard uniformity, axiom of choice, as well as restrictions to disjunction-free formulas of independence of premise, overspill, and underspill principles; the induction axiom is also reserved to disjunction-free formulas. The difference between the nonstandard Dialectica and the uniform Diller-Nahm interpretations is that: the latter operates with single witnessing terms, instead of finite sequences of witnesses; the later's clause for interpreting disjunction, interprets it using an explicit encoding of disjunction via the existential quantifier.

Overall, the reviewer found the article to be satisfactorily detailed and easy to follow.