REVIEW OF "ON IMPLICATIONAL INTERMEDIATE LOGICS AXIOMATIZABLE BY FORMULAS MINIMAL IN CLASSICAL LOGIC: A COUNTER EXAMPLE TO THE KOMORI-KASHIMA PROBLEM" BY YOSHIKI NAKAMURA AND NAOSAKE MATSUDA

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Let IL and CL denote the fragments of intutionistic and classical propositional logic having implication as the sole logical connective. Call a formula (a theorem) α of CL "minimal in CL" if α cannot be obtained by substituting in another formula $\beta \in$ CL a propositional variable p by any formula γ. In a way, α being minimal in CL means that α is apt to be an axiom (schema) of an intermediate logic, since its form is general enough that it cannot be obtained as an instantiation of another more general axiom (schema). IL and CL themselves can be completely axiomatized by such formulas and a question was posed by Komori and Kashima whether one can axiomatize a proper intermediate logic using such a formula.

The authors show that it can be done, by displaying an explicit formula G', a variant of and equivalent to the formula $G := ((a \to b) \to c) \to ((b \to a) \to c) \to c$, which when added to IL produces the intermediate Gödel-Dummett logic. One could not simple take G to be the required formula, because it is not minimal in CL.

The authors also discuss three related open problems.

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