REVIEW OF "PARALLEL POSTULATES AND CONTINUITY AXIOMS: A MECHANIZED STUDY IN INTUITIONISTIC LOGIC USING COQ" BY PIERRE BOUTRY, CHARLY GRIES, JULIEN NARBOUX AND PASCAL SCHRECK (2019)

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The paper studies thirty-four versions of Euclid's parallel postulate, all equivalent over Archimedean planar neutral geometry, in a fully formal way using the Coq proof assistant. Many illustrations, historical remarks, and links to related work, complement the formal development.

Precise relations are proven between the versions of the parallel postulate, using Coq and working over an intuitionistic meta-theory. Namely, thirty-three of the principles can be partitioned into four classes of principles. All of the principles within one class are intuitionistically equivalent over Tarski's neutral geometry assuming decidability of equality of points. Then, a representative of each class is chosen, and their mutual relation is studied. Namely, from the weakest to the strongest:

- Bachmann's Lotschittaxiom, which, in presence of Archimedes' axiom, implies:
- Triangle postulate, which, in presence of Greenberg's axiom (itself implied by Archimedes' axiom), implies:
- Playfair's postulate, which, assuming decidability of line intersection, implies:
- Tarski's parallel postulate.

The reverse implications, from the strongest to the weakest of these principles, hold intuitionistically (without assuming extra principles).