REVIEW OF "A REALIZABILITY INTERPRETATION OF CHURCH'S SIMPLE THEORY OF TYPES" BY ULRICH BERGER AND TIE HOU (2017)

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The paper gives a computational interpretation of proofs of CST which is sound in RCST. CST, the interpreted system, is a version of Church's simple theory of types, without classical logic and the epsilon operator, and with formula-operators for greatest (ν) and least (μ) fixed point that allow to represent induction and co-induction in an elegant way. RCST, the interpreting system, is CST extended with a base type δ containing the terms which are used for realizing CST proofs.

The paper works out in detail the proof of realisability of the fixed-point operators μ and ν, the main novelty of the CST representation of Church's theory.

Although the system is intuitionistic, its intended application is classical.

An advantage of using CST for formalizing mathematics seems to be that certain concepts can be expressed more directly (by using μ and ν) than in other systems equipped with a computational interpretation of proofs (e.g., the ones underlying the proof assistants Coq and Isabelle/HOL), such as the definition of uniform continuity on a compact interval.

The authors might have found it interesting to compare the systems CST and RCST that they use with the versions of Church's theory of types, also extended by the fixed-point operators μ and ν, that appear in research around focusing sequent calculi (for instance, Baelde, D. Least and Greatest Fixed Points in Linear Logic, ACM Transactions on Computational Logic, vol. 13, no. 1, January 2012).